Pearson Edexcel

Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2023
Question Paper Log Number P72464A
Publications Code WFM01_01_MS_2301

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for this paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- * – The answer is printed on the paper
- $\lfloor$ – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general priniciples)

<u>Method mark for solving 3 term quadratic:</u>

1. **Factorisation**
   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$
   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. **Formula**
   Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. **Completing the square**
   Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $x = \ldots$

<u>Method mark for differentiation and integration:</u>

1. **Differentiation**
   Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. **Integration**
   Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

<u>Use of a formula</u>
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1.(a)	$\begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix} = \begin{pmatrix} 2+0+6k & 2k+3+6 \\ -2+0+0 & -2k-9+0 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2+6k & 2k+9 \\ -2 & -2k-9 \end{pmatrix}$	A1cao
		(2)
(b)	det $\mathbf{AB} = (2+6k)(-2k-9)-(-2)(2k+9)$	M1
	det $\mathbf{AB} = 0 \Rightarrow -12k^2 - 54k = 0 \Rightarrow k = ...$	dM1
	$k = -\frac{9}{2}$	A1
		(3)
		(5 marks)

Notes:

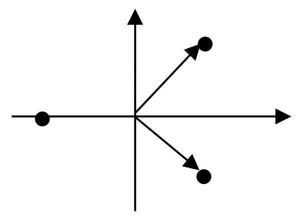
(a)
M1: Obtains a 2×2 matrix with at least two entries correct, unsimplified.
A1cao: Correct matrix with terms simplified.
(b)
M1: Attempts the determinant, be tolerant of minor slips, such as sign slips with the negatives, if the correct "*ad – bc*" form is apparent. They may give the $-(-2)(\ldots)$ as just + 2(…). Accept if seen as part of the attempt at the inverse matrix.
dM1: Expands their determinant to a **quadratic**, sets equal to zero (may be implied) and achieves a value for *k* via correct method (allow if a factor *k* is cancelled, use of formula or calculator (a correct value for their quadratic)).
A1: cso for $-\frac{9}{2}$. Accept as decimal or equivalent fractions, such as $-\frac{54}{12}$. Ignore any reference to the 0 solution.

Question	Scheme	Marks
2.	$(7r-5)^2 = 49r^2 - 70r + 25$	**B1**
	$\sum_{r=1}^{n}(7r-5)^2 = 49\sum_{r=1}^{n} r^2 - 70\sum_{r=1}^{n} r + \sum_{r=1}^{n} 25$ $= 49\times\underline{\frac{n}{6}(n+1)(2n+1)} - 70\times\underline{\frac{n}{2}(n+1)} + 25\underline{\times n}$	**M1** **A1ft**
	$=\frac{n}{6}\left(49(2n^2+3n+1) - 210(n+1) + 150\right)$	**M1**
	$=\frac{n}{6}\left(98n^2 - 63n - 11\right)$	**A1**
	$=\frac{n(7n+1)(14n-11)}{6}$	**A1**
		(6)
		(6 marks)

Notes:

B1: Correct expansion.
M1: Attempts the summations with at least two of the underlined formulae correct.
A1ft: Fully correct application of all three summations. Follow through on their expansion as long as there are 3 terms.
M1: Attempts to factor out at least the factor of n from their **three term** expansion – must have a common factor of n throughout to be able to score this mark which must be extracted from each term. (If the last term is +25, it is M0.) Allow if there are minor slips but the process must be correct.

Alternatively allow this mark for an attempt to expand $\frac{n}{6}(7n+1)\left(An+B\right)$ and compare coefficients with their expanded equation.
A1: Gathers terms appropriately and achieves the correct quadratic. In the alternative approach allow for A =14 and $B = -11$ stated from their comparison.
A1cso: Correct answer from correct work. Any values found from the comparison approach must be substituted back in to achieve the result. Note from a correct unsimplified quadratic to correct answer, A0A1 can be awarded.

Question	Scheme	Marks
	$\mathrm{f}(z) = 4z^3 + pz^2 - 24z + 108$, -3 a root.	
3(a)	$\mathrm{f}(-3) = 0 \Rightarrow 4(-3)^3 + p(-3)^2 - 24(-3) + 108 = 0 \Rightarrow p = ...$	**M1**
	$p = -8$	**A1**
		(2)
(b)	$4z^3 - 8z^2 - 24z + 108 = (z+3)(4z^2 + ...z + 36)$	**M1**
	$= (z+3)(4z^2 - 20z + 36)$	**A1**
	$4z^2 - 20z + 36 = 0 \Rightarrow z = \dfrac{20 \pm \sqrt{400 - 4\times4\times36}}{8} = ...$	**dM1**
	Roots are $-3, \dfrac{5 \pm \mathrm{i}\sqrt{11}}{2}$	**A1**
		(4)
(c)	e.g. Product of complex roots is $\dfrac{36}{4} = 9$, so modulus is $\sqrt{"9"}$ or Modulus is $\sqrt{\left(\dfrac{5}{2}\right)^2 + \left(\dfrac{\sqrt{11}}{2}\right)^2}$	**M1**
	Hence modulus is 3	**A1**
		(2)
(d)	 Complex conjugate pair in correct quadrant for their roots	**M1**
	All three roots correctly positioned.	**A1**
		(2)
		(10 marks)

Notes:

Mark the question as a whole - do not be concerned part labelling.

(a)

M1: A complete method to find the value of *p*. Use of the factor theorem is most direct, look for setting f(−3) = 0 and solving for *p*. May attempt to factor out $(z+3)$ and compare coefficients, e.g.

$f(z) = 4z^3 + pz^2 - 24z + 108 = (z+3)\left(4z^2 + bz + 36\right) \Rightarrow 3b + 36 = -24, 12 + b = p \Rightarrow b = .., p = ...$ or may attempt long division and set remainder equal to zero to find p or variations on these.

A1: For $p = -8$

(b)

Note: Allow marks in (b) for work seen in (a) e.g. via attempts in (a) by long division.

M1: Correct strategy to find a quadratic factor. If factorising, look for correct first and last terms. May use long division, in which case look for the correct first term and attempt to use it - may have been seen in (a).

Question instructs use of algebra so an algebraic method must be seen.

A1: Correct quadratic factor - may have been seen in (a).

dM1: Uses the quadratic formula or completing the square or calculator to find the roots of their quadratic factor (allow for attempts at a quadratic factor via long division which had non-zero remainder). If a calculator is used (no method shown), there must be at least one correct **complex** root for their equation. Factorisation is M0.

A1: Correct roots in simplest form. All three should be included at some point in the solution in (b).

(c)

M1: Any correct method to find the modulus of the complex roots. Most likely to see Pythagoras, but some may deduce from product of roots. They must have complex roots to score the marks in (c).

A1: Modulus 3 only. If -3 is also given as a modulus then score A0.

(d)

Note: Allow the marks in (d) if the i's were missing in their roots in (b) but they clearly mean the correct complex roots on their diagram.

M1: Plots the complex roots as a conjugate pair in the correct quadrants for their roots.

A1: Fully correct diagram with one root on the negative real axis, and the other as a complex pair of roughly the same length in quadrants 1 and 4.

Question	Scheme	Marks
4(a)	(i) $\mathrm{f}'(x) = \underline{Ax^{-5}} + \underline{Bx^{-\frac{9}{2}}}$ oe for at least one power	M1
	$\mathrm{f}'(x) = -\dfrac{-4x^{-5}}{8} + \dfrac{2\times -\frac{7}{2}x^{-\frac{9}{2}}}{7} = \dfrac{1}{2x^5} - \dfrac{1}{x^{\frac{9}{2}}}$ oe	A1
	(ii) Since $\mathrm{f}'(0.25) = 512 - 512 = 0$ the process cannot be carried out as it would require division by zero.	B1
	(iii) $\alpha = 0.15 - \dfrac{\mathrm{f}(0.15)}{\mathrm{f}'(0.15)} = 0.15 - \dfrac{-27.332...}{1484.137...} = ...$	M1
	$= 0.168$ to 3 d.p.	A1cso
		(5)
(b)	e.g. $\dfrac{\mathrm{f}(0.25) - \mathrm{f}(0.15)}{0.25 - 0.15} = \dfrac{\mathrm{f}(0.15) - 0}{0.15 - \alpha}$ or $\dfrac{\alpha - 0.25}{0 - \mathrm{f}(0.25)} = \dfrac{\alpha - 0.15}{0 - \mathrm{f}(0.15)}$ etc	M1
	$\Rightarrow \alpha = 0.15 - \dfrac{0.1\times \mathrm{f}(0.15)}{\mathrm{f}(0.25) - \mathrm{f}(0.15)} = ...$ or $\alpha = \dfrac{0.25\mathrm{f}(0.15) - 0.15\mathrm{f}(0.25)}{\left(\mathrm{f}(0.15) - \mathrm{f}(0.25)\right)} = ...$ etc	M1
	$= 0.15 - \dfrac{0.1\times -27.332...}{5.571... - (-27.332)} = 0.23306... = \text{awrt } 0.233$ (3 d.p.)	A1
		(3)
		(8 marks)

Notes:

(a)(i)

M1: Attempts to differentiate f(x), obtaining the correct power for at least one term.

A1: Correct differentiation, need not be simplified.

(ii)

B1: Correct reason given, accept e.g. "as f'(0.25) = 0" as a minimum and isw after a correct reason is given. Just stating f'(0.25) = 0 is not sufficient, there must be an indication this is the reason why the process cannot be used but accept any indication (such as "not valid") following this.

(iii)

M1: Correct Newton-Raphson process attempted using their derivative or implied by the correct answer from use of a calculator.

A1cso: Correct answer from correct work (derivative must have been correct). Must be 3dp.

(b)

M1: Correct interpolation strategy. Accept any correct statement such as the one shown. They may use e.g. x for $\alpha - 0.15$, in which case the the method will be gained once the correct overall strategy is clear.

M1: Proceeds from a recognisable attempt at interpolation to find a value for α. Not dependent, but they must have attempted to set up a suitable equation in α. If no working is shown accept any value for the root following the suitable statement.

A1: Accept awrt 0.233 following correct working.

Question	Scheme	Marks
5(a)	$\alpha+\beta=-\frac{3}{4}$	B1
	$\alpha\beta=\frac{k}{4}$	B1
		(2)
(b)	$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$	B1
	$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2};=\frac{\left(-\frac{3}{4}\right)^3-3\left(\frac{k}{4}\right)\left(-\frac{3}{4}\right)}{\left(\frac{k}{4}\right)^2}=...$	M1; M1
	$=\frac{36k-27}{4k^2}=\frac{9}{k}-\frac{27}{4k^2}$	A1
		(4)
(c)	Product of roots is $\frac{\alpha\beta}{\alpha^2\beta^2}=\frac{1}{\alpha\beta}=\frac{4}{k}$	B1ft
	Equation is $x^2-\left(\frac{36k-27}{4k^2}\right)x+\frac{4}{k}=0$	M1
	$4k^2x^2-(36k-27)x+16k=0$	A1
		(3)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\alpha+\beta$

B1: Correct expression for $\alpha\beta$

(b)

B1: Combines the fractions correctly.

M1: For a correct identity for the sum of cubes.

M1: Substitutes their values for $\alpha+\beta$ and $\alpha\beta$ into their equation for sum of $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$ (not dependent, so there may be a slip in the identity used for $\alpha^3+\beta^3$).

A1: Correct expression in terms of *k* in a simplified form - e.g. either form as shown in scheme.

(c)

B1ft: Correct product of roots in terms of *k*, or follow through $\frac{1}{\text{their }\alpha\beta}$ from part (a).

M1: Applies $x^2 - (\text{their sum of roots})x + \text{their product of roots}\left(=0\right)$. Allow without the "=0" for this mark.

A1: Correct equation, as shown or an integer multiple thereof. Accept equivalents for the *x* term (e.g. $4k^2x^2 + (27-36k)x + 16k = 0$. Must include the "=0".

Question	Scheme	Marks
6(a)	$a=5$	B1
		(1)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{20}{x^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\sqrt{5}}{t^2}\div 2\sqrt{5}=-\frac{1}{t^2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ oe	B1
	Gradient of normal is $\frac{-1}{\text{"}-1/t^2\text{"}}=t^2$	M1
	Normal is $y-\frac{2\sqrt{5}}{t}=t^2\left(x-2t\sqrt{5}\right)$	M1
	$\Rightarrow ty-2\sqrt{5}=t^3x-2t^4\sqrt{5}\Rightarrow ty-t^3x-2\sqrt{5}\left(1-t^4\right)=0$ *	A1*
		(4)
(c)	$cy-c^3x-2\sqrt{5}\left(1-c^4\right)=0$ passes through $\left(-\frac{\sqrt{5}}{c},-4c\sqrt{5}\right)$ $\Rightarrow -4c^2\sqrt{5}+c^2\sqrt{5}-2\sqrt{5}\left(1-c^4\right)=0$	M1
	$\Rightarrow 2c^4-3c^2-2=0$ (oe)	A1
	$\Rightarrow c^2=\frac{3\pm\sqrt{9-4\times2\times-2}}{4}=\ldots\left(2,-\frac{1}{2}\right)$	dM1
	$c^2>0\Rightarrow c^2=2\Rightarrow c=\pm\sqrt{2}$	A1
		(4)
		(9 marks)

Notes:

(a)

B1: Correct value stated.

(b)

B1: Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, or any equivalent correct expression including it, such as $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P. Allow with a instead of 5 for this mark, so score for e.g. $m_N=\frac{4at^2}{20}$.

M1: Uses their normal (changed from tangent) gradient and P to find the equation of the tangent. Look for $y-\frac{2\sqrt{5}}{t}=\text{"}m_n\text{"}\left(x-t\sqrt{5}\right)$. If using $y = mx + c$ they must proceed as far as finding c.

A1*: Correct equation achieved from correct working with intermediate step.

(c)

M1: Substitutes the parameter for *A* into the normal equation and attempts to substitute the coordinates of *B* to obtain an equation in one variable. Allow if there are slips during substitution. **A1:** Correct quadratic in c^2 need not be simplified. **dM1:** Solves their (at least two term) quadratic in c^2 to find a value for at least c^2 **A1:** Deduces correct values. Both required. Ignore reference to any complex roots.		
Alts		
(c)	$cy - c^3x - 2\sqrt{5}\left(1-c^4\right) = 0$ intersects *H* again $\Rightarrow \dfrac{20}{x}c - c^3x - 2\sqrt{5}\left(1-c^4\right) = 0$ or $cy - \dfrac{20}{y}c^3 - 2\sqrt{5}\left(1-c^4\right) = 0$ $\Rightarrow c^3x^2 + 2\sqrt{5}\left(1-c^4\right)x - 20c = 0$ or $cy^2 - 2\sqrt{5}\left(1-c^4\right)y - 20c^3 = 0$	**M1** **A1**
	$\Rightarrow \left(c^3x + 2\sqrt{5}\right)\left(x - 2c\sqrt{5}\right) = 0$ or $\left(cy - 2\sqrt{5}\right)\left(y + 2\sqrt{5}c^3\right) = 0$ $\left(x = 2c\sqrt{5} \text{ is } A \text{ so}\right)$ for *B* $x = -\dfrac{2\sqrt{5}}{c^3} = -\dfrac{\sqrt{5}}{c} \Rightarrow c = \ldots$ $\left(\text{or } y = \dfrac{2\sqrt{5}}{c} \text{ is } A \text{ so}\right)$ for *B* $y = -2\sqrt{5}c^3 = -4c\sqrt{5} \Rightarrow c = \ldots$	**M1**
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	**A1**
		(4)
Notes:		
(c) **M1:** Substitutes parameters for *A* and equation for *H* into normal to obtain a quadratic in *x*. **A1:** Correct quadratic in *x* or *y* **M1:** Solves the quadratic in *x* or *y*, identifies correct solution and equates to the relevant coordinate of *B* and solves for *c* **A1:** Deduces correct values. Both required.		
(c)	$m_{AB} = \dfrac{\dfrac{2\sqrt{5}}{c} - \left(-4c\sqrt{5}\right)}{2\sqrt{5}c - \left(-\dfrac{\sqrt{5}}{c}\right)}$	**M1**
	$m_{AB} = \dfrac{2+4c^2}{2c^2+1} = 2$	**A1**
	From (b), normal at *A* has gradient $(t^2 =)c^2 \Rightarrow c^2 = 2$	**M1**
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	**A1**
		(4)
Notes:		

(c)
M1: Attempts the gradient of *AB*.
A1: Correct gradient need not be simplified.
M1: Finds/deduces the gradient of the normal at *A* and sets equal to their gradient of *AB.*
A1: Deduces correct values. Both required.

Question	Scheme	Marks
7(i)(a)	Reflection or in the line $y=-x$	**M1**
	Reflection in the line $y=-x$	**A1**
		(2)
(b)	$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ or $6\times\begin{pmatrix} \pm\cos 240° & \pm\sin 240° \\ \pm\sin 240° & \pm\cos 240° \end{pmatrix}$	**M1**
	$\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}$	**A1**
		(2)
(c)	$\mathbf{R}=\mathbf{QP}=\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}=\ldots$	**M1**
	$=\begin{pmatrix} -3\sqrt{3} & 3 \\ 3 & 3\sqrt{3} \end{pmatrix}$ **QP** correctly found	**A1**
		(2)
(ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}\Rightarrow\begin{pmatrix} -2\lambda+2\sqrt{3} \\ 2\lambda\sqrt{3}+2 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$	**M1**
	$-2\lambda+2\sqrt{3}=4\lambda$ or $2\lambda\sqrt{3}+2=4$	**A1**
	$\Rightarrow 6\lambda=2\sqrt{3}\Rightarrow\lambda=\ldots$ or $2\sqrt{3}\lambda=2\Rightarrow\lambda=\ldots$	**dM1**
	$\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ oe	**A1**
	Both $-2\lambda+2\sqrt{3}=4\lambda$ and $2\lambda\sqrt{3}+2=4$ solved leading to $\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	**A1**
		(5)
		(11 marks)

Notes:

(a)
M1: Identifies the transformation as a reflection or identifies the correct line of reflection.

A1: Fully correct description, with the equation of the line of reflection or suitable description (e.g. in the line through angle 135° with the positive *x*-axis). Ignore any references to a centre of reflection.

(b)

M1: Either the correct matrix for the rotation (with trig ratios evaluated) or an attempt at scaling a matrix of form shown by a factor 6 (need not evaluate ratio) – if no trig ratios seen this may be implied by the exact values in the right places. The correct answer implies the M.

A1: Correct matrix.

(c)

M1: Attempts to multiply **Q** and **P** in the correct order.

A1: QP correct

(ii)

M1: Attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}$ and sets equal to $\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$. Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.

A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.

A1: Correct value for λ from at least one equation and isw if incorrectly simplified (allow if their second equation does not concur).

A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Alt (ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}\Rightarrow\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\frac{1}{-4-12}\begin{pmatrix} 2 & -2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$ $\Rightarrow\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=-\frac{1}{16}\begin{pmatrix} 8\lambda-8\sqrt{3} \\ -8\lambda\sqrt{3}-8 \end{pmatrix}$	**M1**
	$2\lambda=\sqrt{3}-\lambda$ or $2=\lambda\sqrt{3}+1$	**A1**
	$\Rightarrow 3\lambda=\sqrt{3}\Rightarrow\lambda=...$ or $\sqrt{3}\lambda=1\Rightarrow\lambda=...$	**dM1**
	$\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	**A1**
	Both $2\lambda=\sqrt{3}-\lambda$ and $2=\lambda\sqrt{3}+1$ solved leading to $\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	**A1**
		(5)

Notes:

M1: Correct attempt at inverse, attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}^{-1}\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$ and sets equal to $\begin{pmatrix} \lambda \\ 1 \end{pmatrix}$.

Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.

A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.

A1: Correct value for λ from at least one equation (allow if their second equation does not concur).

A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Question	Scheme	Marks
8(a)	$y^2 = 4ax, y = k \Rightarrow P = \left(\frac{k^2}{4a}, k\right)$	**B1**
	Either $PS = \frac{k^2}{4a} + a = ...$ or $PS^2 = \left(\frac{k^2}{4a} - a\right)^2 + k^2 = ... \Rightarrow PS = ...$	**M1**
	$PS = \frac{k^2 + 4a^2}{4a}$ *	**A1***
		(3)
(b)	Gradient of l_2 is $\frac{k}{\frac{k^2+4a^2}{4a}} = \frac{4ak}{k^2+4a^2}$ oe	**B1**
	$l_2 : y = \frac{4ak}{k^2+4a^2}(x+a) \Rightarrow y = \frac{4ak}{k^2+4a^2} \times (0+a) = ..$	**M1**
	$y\big\|_{x=0} = \frac{4a^2k}{k^2+4a^2}$ *	**A1***
		(3)
(c)	Area $OSP = \frac{1}{2} \times a \times k$	**B1**
	Area $BPA = \frac{1}{2} \times \frac{k^2+4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2+4a^2}\right)$ $\left(= \frac{k^3}{8a}\right)$	**M1**
	$\frac{\text{Area } BPA}{\text{Area } OSP} = \frac{\frac{k^2+4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2+4a^2}\right)}{ak} = 4k^2$	**M1**
	$\Rightarrow k^3 + 4a^2k - 4a^2k = 16a^2k^3 \Rightarrow a = ...$	**dM1**
	$a = \frac{1}{4}$	**A1**
		(5)
		(11 marks)

Notes:

(a)
B1: Correct x coordinate at P found. May be seen on diagram.
M1: For a full method to find an expression for PS. Either use of focus-directrix property or may use Pythagoras with their coordinates.

A1*: Reaches the correct expression with a suitable intermediate step and no errors seen. If using Pythagoras the suitable step must be one with brackets expanded before factorising again.

(b)

B1: Correct expression for the gradient of l_2 given or implied by working. Need not be simplified. If using a similar triangles approach this may be scored for e.g. $\frac{k^2+4a^2}{4a} \div k = \frac{a}{y}$ or $k = \left(\frac{k^2+4a^2}{4a}\right)m$

M1: Full method to find the y intercept, e.g. by forming the equation of the line and substituting $x = 0$

May use $y - k = \frac{4ak}{k^2+4a^2}\left(x - \frac{k^2}{4a}\right) \Rightarrow y = k + \frac{-k^3}{k^2+4a^2}$

A1*: Reaches correct answer with no errors seen.

(c)

NB: If a value is chosen for k (or $k = 2a$) used, all marks are available, score for the relevant correct expressions/methods with their value.

B1: Correct area of OSP stated or implied. Note that if they go direct to ratios, the $\frac{1}{2}$ may not be seen (as it cancels with that in BPA)

M1: Correct method for the area of the triangle BPA. Allow sign slips if the method is clear (e.g. $\frac{k^2}{4a} - "-a" = \frac{k^2}{4a} - a$ if it is clear BP is meant). Allow if the negative of the area is found.

M1: Applies the ratio correctly to the problem.

dM1: Attempts to solve their equation.

A1: Correct answer. Allow if the negative of the area was found and later made positive as recovery.

Question	Scheme	Marks
9	For $n = 1$, $\sum_{r=1}^{1}\log(2r-1)=\log(2-1)=\log 1$ and $\log\left(\frac{(2\times1)!}{2^1 1!}\right)=\log 1$ So true for $n = 1$	B1
	(Assume the result is true for $n = k$, so $\sum_{r=1}^{k}\log(2r-1)=\log\left(\frac{(2k)!}{2^k k!}\right)$ Then) $\sum_{r=1}^{k+1}\log(2r-1)=\log\left(\frac{(2k)!}{2^k k!}\right)+\log(2(k+1)-1)$	M1
	$=\log\left(\frac{(2k)!}{2^k k!}\times(2k+1)\right)$	M1
	$=\log\left(\frac{(2k+1)!}{2^k k!}\times\frac{2k+2}{2k+2}\right)=\log\left(\frac{(2k+2)!}{2^k\times 2(k+1)!}\right)$	M1
	$=\log\left(\frac{(2k+2)!}{2^{k+1}(k+1)!}\right)$	A1
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all $n\in\mathbb{N}$ by induction.	A1
		(6)
		(6 marks)

Notes:

(a)

B1: Checks the result for $n = 1$. Must see **both** sides (possibly in one line) identified as log1 or 0 but may not see much more than this.

M1: Makes the inductive assumption (may be implied) and applies it to the question by adding the $(k+1)^{\text{th}}$ term to the expression for the sum to k terms. Allow if there are minor slips (e.g. a missing factorial) if the intent is clear.

M1: Attempts to combine or split log terms appropriately. Not dependent, so may be scored if the wrong term is added in the previous M as long it is a log term.

M1: Introduces the relevant cancelling factors to achieve the $(2k + 2)!$ term. The introduction of the factors must shown or implied in an intermediate step. Alternatively, may decompose from the $k+1$ statement to achieve the same intermediate expression.

A1: Achieves correct expression from correct work (or correctly shows equivalence).

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

NB Allow the M's and first A if n is used throughout but the steps are correct, but must have used a different variable for the final A.

Alt steps if splitting logs:	$\sum_{r=1}^{k+1}\log(2r-1)=\log\left(\frac{(2k)!}{2^k k!}\right)+\log(2k+1)$ **M1** $=\log(2k)!-\log(2^k k!)+\log(2k+1)$ **M1** $=\log(2k+1)!-\log\left(\frac{2^{k+1}(k+1)!}{2\times(k+1)}\right)=\log\left(\frac{(2k+1)!(2k+2)}{2^k(k+1)!}\right)$ **M1**

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom